Stability Criterion for Passive Magnetically Anchored Rate Dampers

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Nomenclature

b	= damping coefficient of the damper along each axis
I_x, I_y, I_z	= yaw, roll, and pitch moments of inertia of the spacecraft
I_x', I_y', I_z'	= yaw, roll, and pitch moments of inertia of the inner sphere of the damper
K_1, K_2	= factors ($0 \le K_1$, $K_2 \le 1.0$) depending on the inclination of the spacecraft orbit to account for the reduced magnetic and damping torques on the damper in a nonpolar orbit – for a polar orbit, $K_1 = K_2 = 1.0$
T	= magnetic torque per unit angular displacement on the damper for a polar orbit at the equator
$\dot{\eta}$	= angular orbital rate of the spacecraft
θ_r, θ_y	= roll and yaw angles of the spacecraft
θ_r', θ_y'	= roll and yaw angles of the damper
$(\dot{\ }), (\ddot{\ })$	= first and second derivatives with respect to time

Theme

MAGNETICALLY anchored rate dampers have proved to be very useful passive elements for damping the oscillations of gravity-gradient-stabilized spacecraft. The present work identifies a set of conditions under which a limit cycle can be generated by the damper. The results of a linear analysis, nonlinear simulations, and eigenvalue searches are now presented.

Contents

Introduction

A magnetically anchored rate damper essentially consists of a permanent magnet placed inside a spherical copper shell. The shell is fixed in the spacecraft and is lined inside with a diamagnetic material. The lining suspends the magnet in the shell and the magnet is free to rotate. When the magnet moves inside the shell, damping torques opposing the motion are generated due to the eddy currents in the shell. In some designs, the magnet is fixed to an inner sphere which is contained in the outer shell with a viscous liquid separating the inner sphere from the outer shell. In such a design, damping torques are generated due to fluid friction as well as eddy currents. In general, the magnet stays aligned with the Earth's magnetic field. Any oscillation of the spacecraft causes relative motion between the magnet and the outer shell and hence is damped out.[1,2,3]

Once the spacecraft has stabilized, the relative motion, however, continues. This is because the magnet rotates with the Earth's magnetic field and the damping torques become disturbance torques on the spacecraft. For an equatorial orbit of the spacecraft, there is very little disturbance torque and the damping torques exist only in the roll and yaw axes of the spacecraft. For a polar orbit, the disturbance torque is only along the pitch axis and all three axes experience damping torques as sinusoidal functions of the orbit position. In either of these two extreme positions, the stability is unconditional and no limit cycles can be generated.

In a nonpolar and nonequatorial orbit, the disturbance and damping torques are present in all three axes of the spacecraft. The disturbance torque about the roll axis imparts a momentum to the spacecraft which is out of phase with the momentum unloaded through the gravity-gradient torques. This causes a limit cycle in the spacecraft as evidenced by local instabilities of the equations of motion linearized about zero attitude errors.

Analysis

A detailed set of equations of motion for the spacecraft and the damper is given in the original paper. A simplified form of those equations will now be considered. The pitching motion of a gravity-gradient-stabilized spacecraft is essentially uncoupled from the motion in the roll-yaw plane, and is not considered. The equations of motion for small angular rotations of the spacecraft and the damper in the roll-yaw plane can be written as follows:

$$I_x\ddot{\theta}_y + b\dot{\theta}_y + \dot{\eta}(I_z - I_y - I_x)\dot{\theta}_r + (\dot{\eta})^2(I_z - I_y)\theta_y - b\dot{\theta}_y' = 0 \quad (1)$$

$$I_y\ddot{\theta}_r + \dot{\eta}(I_x + I_y - I_z)\dot{\theta}_y + b\dot{\theta}_r + 4(\dot{\eta})^2(I_z - I_x)\theta_r - b\dot{\theta}_r - \dot{b}_r' = 0 \quad (2)$$

$$I_x'\ddot{\theta}_y' + b\dot{\theta}_y' + \dot{\eta}(I_z' - I_y' - I_x')\dot{\theta}_r' + K_2T\theta_y' - b\dot{\theta}_y = 0 \quad (3)$$

$$I_y'\ddot{\theta}_r' + \dot{\eta}(I_x' + I_y' - I_z')\dot{\theta}_y' + K_1b\dot{\theta}_r' + K_2T\theta_r' - K_1b\dot{\theta}_r = 0 \quad (4)$$

The terms involving the parameters K_1 and K_2 arise due to the motion of the magnet as it follows the Earth's magnetic field. These parameters are actually functions of the orbit position and hence time varying. In this analysis average values of these time-dependent functions over one orbit have been used to obtain an autonomous set of equations. The parameters K_1 and K_2 are thus the averaged values of the true time-dependent coefficients. The exact form of the time-dependent coefficients is given in the original paper. Neglecting some small terms, a simple Routhian analysis of Eqs. (1-4) yields the following stability criterion.

$$\begin{aligned} & I_xI_yT^2K_2^2[bI_xI_yK_2T(1+K_1)][b(\dot{\eta})^2\{4I_x(I_z-I_x) \\ & + (I_x+I_y-I_z)^2\}K_2(1+K_1)T] - I_xI_yb^2K_1[b(\dot{\eta})^2 \\ & \times\{4I_x(I_z-I_x) + (I_x+I_y-I_z)^2\}K_2(1+K_1)T]^2 \\ & - [(\dot{\eta})^2\{4I_x(I_z-I_x) + (I_x+I_y-I_z)^2\}K_2^2T^2] \\ & \times[bI_xI_yK_2T(1+K_1)]^2 > 0 \end{aligned} \quad (5)$$

The stability criterion given in Eq. (5) shows that a local instability is possible for certain values of the parameters I_X, I_Y, I_X', I_Y', b, T, K_1, and K_2.

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Table 1 Mass properties and orbit parameters

Weight:	9200 lbs
Moments of inertia:	
Yaw	$14200 + 1.52M$ slug-ft^2
Roll	$29000 + 6.94M$ slub-ft^2
Pitch	$29000 + 8.51M$ slug-ft^2
(M = total mass added to the spacecraft for changing the moments of inertia, lbs)	
Products of inertia:	0.0
Orbit altitude:	270 naut. mi
Orbit inclination:	28.8 deg

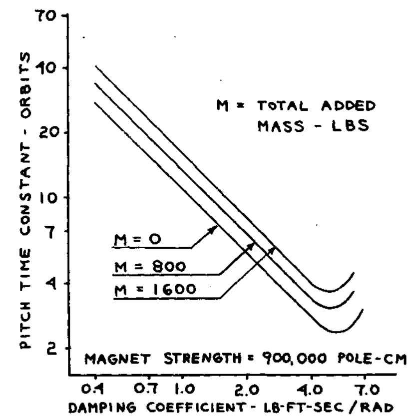

Fig. 1 Pitch time constant vs damping coefficient.

Results

The existence of this instability was first noticed during a search of the eigenvalues of the fully coupled linearized equations involving motion in all three axes. The parameters varied were the damping coefficient, the strength of the damper magnet, and the moments of inertia of the spacecraft. The nominal mass properties and orbit parameters of the spacecraft are shown in Table 1.

The results of this eigenvalue search for a given magnet is shown in Figs. 1 and 2. The moments of inertia of the spacecraft were altered by adding a total mass of M pounds to the spacecraft. This variation is shown in Table 1. As mentioned earlier, the pitching motion is essentially uncoupled from the roll and yaw motions. This is the reason behind the smooth variation of the pitch damping time constant as shown in Fig. 1. There exists a lower limit for the time constant which is evident in that figure. This happens because, at high damping levels, the spacecraft pulls the magnet away from the Earth's magnetic field.

The variation of the damping time constants for the roll-yaw motion of the fully coupled equations is shown in Fig. 2.

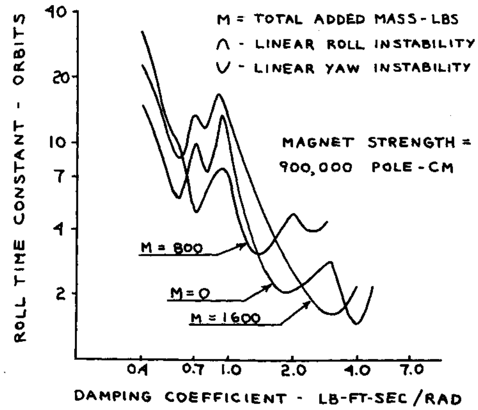

Fig. 2 Roll time constant vs damping coefficient.

The time constants shown in this figure correspond to zero-roll bias angle. It can be seen that the time constants have an oscillatory nature. The motion is undamped at the peaks of the curves. However, if small roll bias angles are incorporated, the peaks either shift, or are smoothed. It was found that at these peaks the inequality (5) is not satisfied.

The problem associated with this local instability was also evident in three axis nonlinear digital simulations. In one simulation, corresponding to a point away from a peak of Fig. 2, the spacecraft captured and stabilized in approximately 100 hours. With the same initial conditions and double the damping level, the spacecraft has not captured in more than 150 hours when the run was terminated. This second simulation corresponded to a point near a peak of the curves in Fig. 2.

Conclusions

Points of local linear instability exist in the dynamics of spacecraft with magnetic dampers. Usually the instability vanishes about nonzero bias values of the attitude errors of the spacecraft. These nonzero stable attitude biases in conjunction with gravity-gradient torques cause the spacecraft to exhibit a limit cycle. This limit cycle is at a frequency close to the orbital frequency. In such a situation, the spacecraft does not settle because the frequencies of the majority of the disturbing torques are also at the same frequency.

References

[1] Foulke, H.F., Siegel, S.H., and Das, A., "A Stability Criterion for Passive Magnetically Anchored Rate Dampers," AIAA/ASS Astrodynamics Conference, Nassau, Bahamas, July, 1975.

[2] "Final Design Report on the Naval Research Laboratory Semi-Passive Stabilization Subsystem," GE Document No. 69SD4342, Aug. 1969, prepared under Contract No. N001783-69-C-0019 for the U.S. Naval Research Laboratory.

[3] "Final Report on the Passive Stabilization of the Long Duration Exposure Facility (LDEF)," GE Document No. 74SD4264, Nov. 1974, prepared under Contract No. NAS 1-13440 for the Langley Research Center, NASA.